

Identification of Aerodynamic Coefficients for a UAV

*For the
Quarterly Review of the NASA/FAA Joint University
Program for Air Transportation Research*

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Introduction

- Equations of Motion
- 6-DOF Modeling
- Aerodynamic Coefficients
- System Identification
- Results
- Future Issues
- Questions

Equations of Motion

- Force equations: $F_B + Bmg = \frac{d}{dt_I}(mv_{abs})$
- Moment equations $T_B = \frac{d}{dt_I}(H_B)$
- Kinematic equations $\dot{\Phi} = \xi(\Phi) \cdot \omega$

Governing Equations

- 3 force equations

$$\dot{U} = rV - qW - g_0 \sin \theta + \frac{F_x}{m}$$

$$\dot{V} = -rU + pW - g_0 \sin \phi \cos \theta + \frac{F_y}{m}$$

$$\dot{W} = qU - pV - g_0 \cos \phi \cos \theta + \frac{F_z}{m}$$

- 3 moment equations

$$\dot{p} = (c_1 r + c_2 p)q + c_3 \bar{L} + c_4 N$$

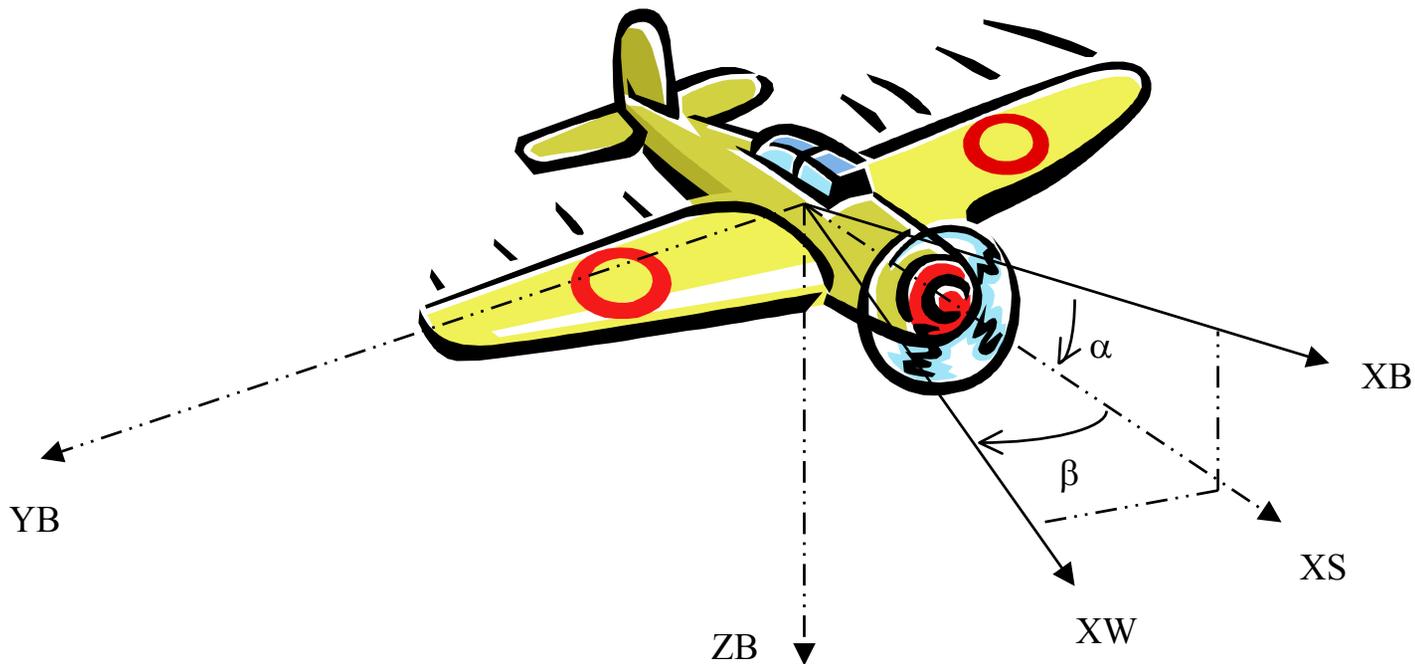
$$\dot{q} = c_5 pr - c_6 (p^2 - r^2) + c_7 M$$

$$\dot{r} = (c_8 p - c_2 r)q + c_4 \bar{L} + c_9 N$$

Aerodynamic Coefficients (Force and Moment Equations)

- Drag: $D = \bar{q}SC_D$
- Lift: $L = \bar{q}SC_L$
- Side force: $Y = \bar{q}SC_Y$
- Rolling moment: $\bar{L} = \bar{q}SbC_{\bar{L}}$
- Pitching moment: $M = \bar{q}S\bar{c}C_M$
- Yawing moment: $N = \bar{q}S\bar{c}C_N$

Wind vs. Body Frame



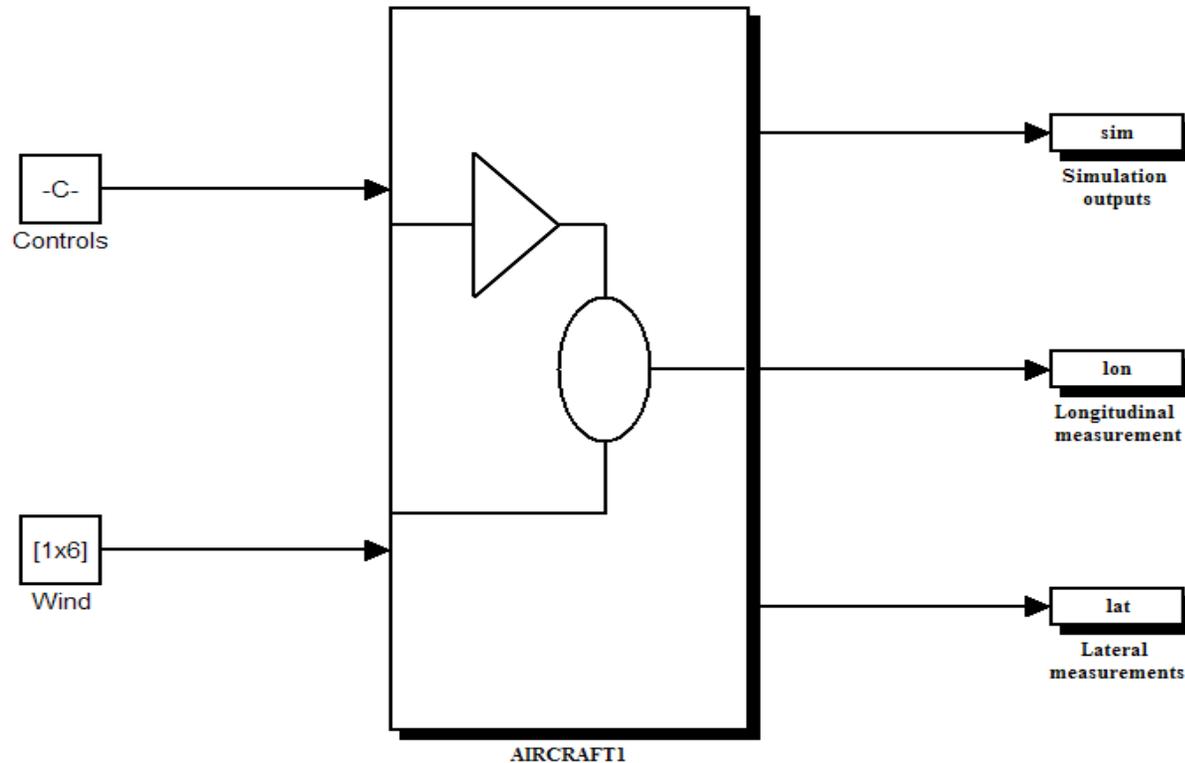
Wind Axes Coordinates (Force Equations)

$$F_B = \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} = \begin{bmatrix} F_{XA} \\ F_{YA} \\ F_{ZA} \end{bmatrix} + \begin{bmatrix} F_{XT} \\ F_{YT} \\ F_{ZT} \end{bmatrix} \quad S \cdot F_B = S \cdot \begin{bmatrix} F_X \\ F_Y \\ F_Z \end{bmatrix} = F_{WA} + F_{WT} = \begin{bmatrix} -D \\ Y \\ -L \end{bmatrix} + S \cdot F_{BT}$$

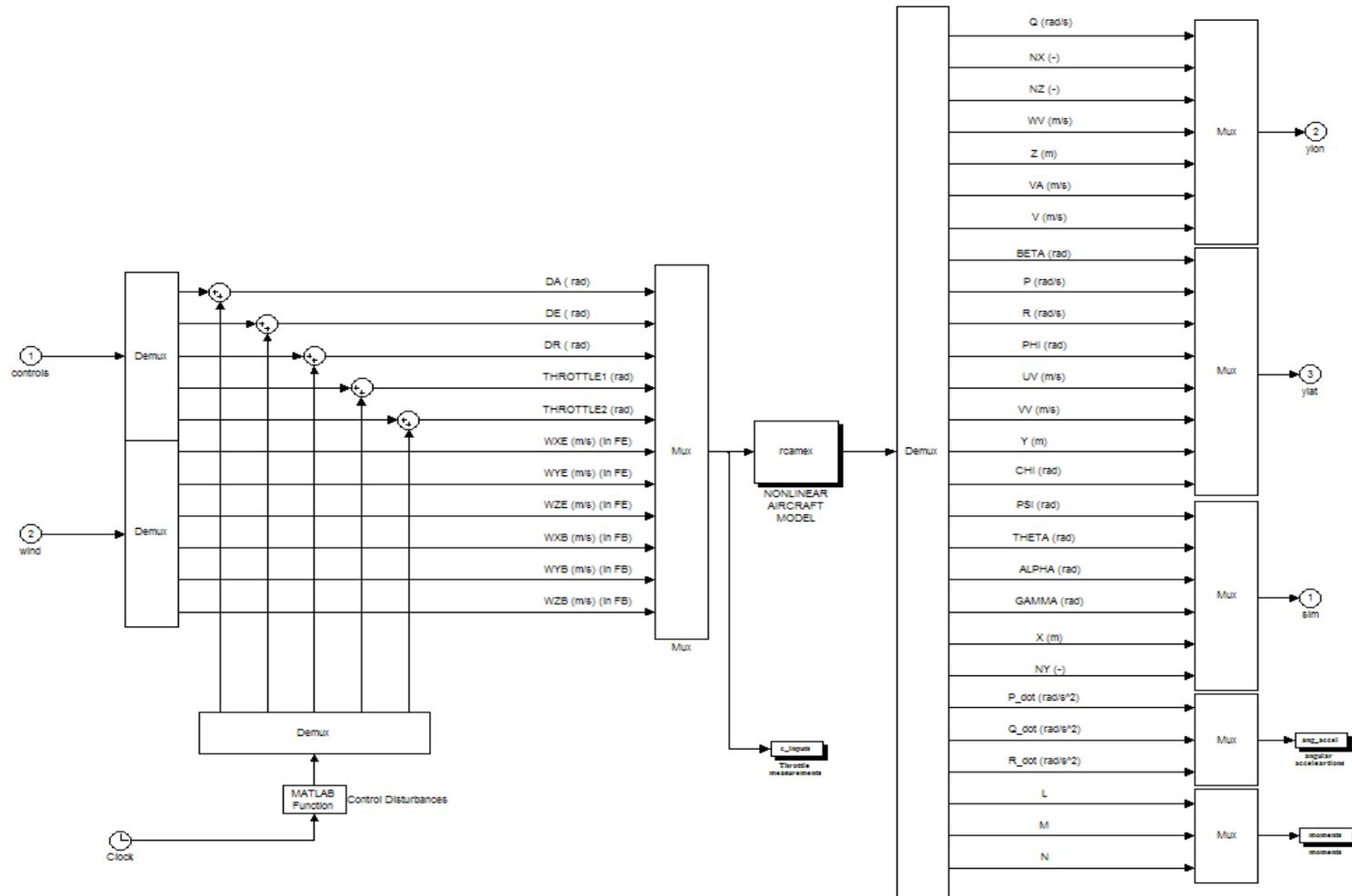
$$S \cdot \begin{bmatrix} F_X \\ F_Y \\ F_Z \end{bmatrix} - S \cdot F_{BT} = \begin{bmatrix} -D \\ Y \\ -L \end{bmatrix}$$

$$F_{BT} = \begin{bmatrix} T \\ 0 \\ 0 \end{bmatrix} \quad S = \begin{bmatrix} \cos \alpha \cdot \cos \beta & \sin \beta & \sin \alpha \cdot \cos \beta \\ -\cos \alpha \cdot \sin \beta & \cos \beta & -\sin \alpha \cdot \sin \beta \\ -\sin \alpha & 0 & \cos \alpha \end{bmatrix}$$

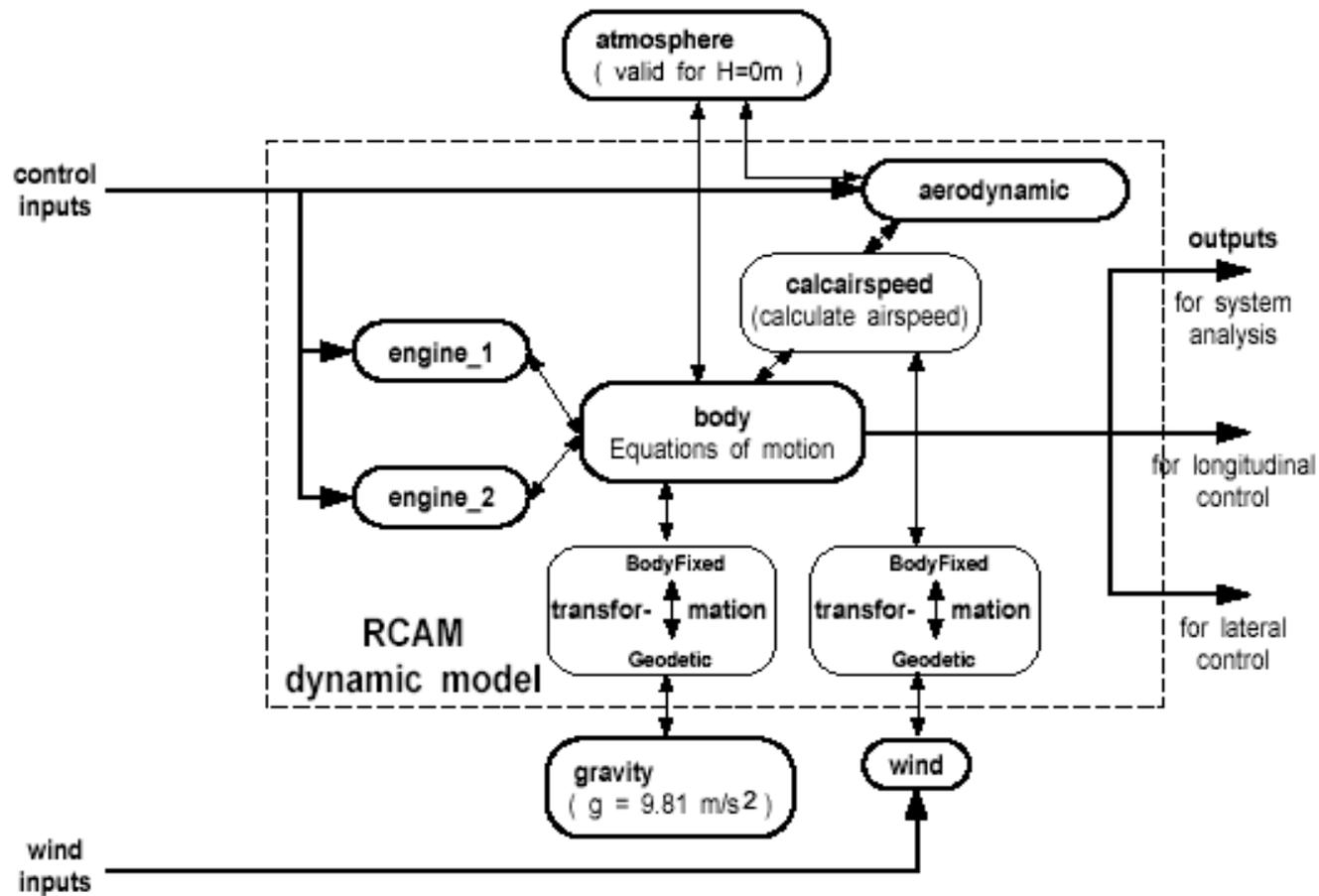
6-DOF Aircraft Model (Simulink)



6-DOF Aircraft Model (Simulink)



Dynamic Aircraft Model



Aerodynamic Coefficients

Force coefficients (wind frame)

$$\begin{bmatrix} -D \\ Y \\ -L \end{bmatrix} = \begin{bmatrix} C_{D0} + C_{D\alpha}(\alpha) + C_{D\alpha^2}(\alpha^2) \\ C_{Y\alpha}(\beta) + C_{Y\delta r}(\delta r) \\ C_{L0} + C_{L\alpha}(\alpha) + C_{Lq}(q/V_a) + C_{L\delta e}(\delta e) \end{bmatrix}$$

Moment coefficients (body frame)

$$\begin{bmatrix} \bar{L} \\ M \\ N \end{bmatrix} = \begin{bmatrix} C_{\bar{L}\beta}(\beta) + C_{\bar{L}p}(p/V_a) + C_{\bar{L}r}(r/V_a) + C_{D\delta\alpha}(\delta\alpha) + C_{\bar{L}\delta r}(\delta r) \\ C_{M0} + C_{M\alpha}(\alpha) + C_{Mq}(q/V_a) + C_{M\delta e}(\delta e) \\ C_{N\beta}(\beta) + C_{N\beta\alpha}(\beta\alpha) + C_{Np}(p/V_a) + C_{Nr}(r/V_a) + C_{N\delta r}(\delta r) \end{bmatrix}$$

Measurement of Aerodynamic Coefficients

Force equations: (wind frame)

$$\begin{bmatrix} F_X(\cos \alpha \cdot \cos \beta) + F_Y(\sin \beta) + F_Z(\sin \alpha \cdot \cos \beta) - T(\cos \alpha \cdot \cos \beta) \\ -F_X(\cos \alpha \cdot \sin \beta) + F_Y(\cos \beta) - F_Z(\sin \alpha \cdot \sin \beta) + T(\cos \alpha \cdot \sin \beta) \\ -F_X(\sin \alpha) + F_Z(\cos \alpha) + T(\sin \alpha) \end{bmatrix} = \begin{bmatrix} -D \\ Y \\ -L \end{bmatrix}$$

Moment equations: (body frame)

$$\begin{bmatrix} \dot{p}I_{xx} + qr(I_{zz} - I_{yy}) + (pq + \dot{r})I_{xz} \\ \dot{q}I_{yy} + rp(I_{xx} - I_{zz}) + (p^2 - r^2)I_{xz} \\ \dot{r}I_{zz} + pq(I_{yy} - I_{xx}) + (qr - \dot{p})I_{xz} \end{bmatrix} = \begin{bmatrix} \bar{L} \\ M \\ N \end{bmatrix}$$

Measurement of Aerodynamic Coefficients

$$\begin{bmatrix} F_X(\cos \alpha \cdot \cos \beta) + F_Y(\sin \beta) + F_Z(\sin \alpha \cdot \cos \beta) - T(\cos \alpha \cdot \cos \beta) \\ -F_X(\cos \alpha \cdot \sin \beta) + F_Y(\cos \beta) - F_Z(\sin \alpha \cdot \sin \beta) + T(\cos \alpha \cdot \sin \beta) \\ -F_X(\sin \alpha) + F_Z(\cos \alpha) + T(\sin \alpha) \end{bmatrix} = \begin{bmatrix} -D \\ Y \\ -L \end{bmatrix}$$

$$\begin{bmatrix} -D \\ Y \\ -L \end{bmatrix} = \begin{bmatrix} C_{D0} + C_{D\alpha}(\alpha) + C_{D\alpha^2}(\alpha^2) \\ C_{Y\alpha}(\beta) + C_{Y\delta r}(\delta r) \\ C_{L0} + C_{L\alpha}(\alpha) + C_{Lq}(q/V_a) + C_{L\delta e}(\delta e) \end{bmatrix}$$

Measurement of Aerodynamic Coefficients

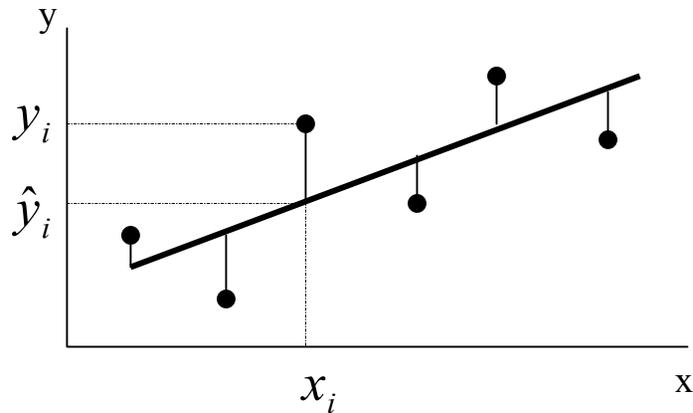
$$\begin{bmatrix} \dot{p}I_{xx} + qr(I_{zz} - I_{yy}) + (pq + \dot{r})I_{xz} \\ \dot{q}I_{yy} + rp(I_{xx} - I_{zz}) + (p^2 - r^2)I_{xz} \\ \dot{r}I_{zz} + pq(I_{yy} - I_{xx}) + (qr - \dot{p})I_{xz} \end{bmatrix} = \begin{bmatrix} \bar{L} \\ M \\ N \end{bmatrix}$$

$$\begin{bmatrix} \bar{L} \\ M \\ N \end{bmatrix} = \begin{bmatrix} C_{\bar{L}\beta}(\beta) + C_{\bar{L}p}(p/V_a) + C_{\bar{L}r}(r/V_a) + C_{D\delta\alpha}(\delta\alpha) + C_{\bar{L}\delta r}(\delta r) \\ C_{M0} + C_{M\alpha}(\alpha) + C_{Mq}(q/V_a) + C_{M\delta e}(\delta e) \\ C_{N\beta}(\beta) + C_{N\beta\alpha}(\beta\alpha) + C_{Np}(p/V_a) + C_{Nr}(r/V_a) + C_{N\delta r}(\delta r) \end{bmatrix}$$

System Identification

- **Goal:** the prediction of system outputs, given the system state and applied inputs.
- **Usefulness:** allows for the control of that system.

Method of Least Squares



$$\text{Model: } \hat{y}_i = \hat{A}_0 + \hat{A}_1 x_i$$

The deviation of the observed value y is: $y_i - \hat{y}_i$

For the L.S. Method the sum of the squares is minimized

$$SSE = \sum_{i=1}^n (y_i - \hat{y}_i)^2 = \sum_{i=1}^n [y_i - (\hat{A}_0 + \hat{A}_1 x_i)]^2$$

Method of Least Squares

Linear model: $Y = A_1 + A_1x_1 + \dots + A_kx_k + \varepsilon$

By making n independent observations, y_1, y_2, \dots, y_n , on Y .

We can write the observations y_i as:

$$Y = A_1 + A_1x_{i1} + \dots + A_kx_{ik} + \varepsilon_i$$

Define:

$$Y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} \quad X = \begin{bmatrix} x_0 & x_{11} & \cdots & x_{1k} \\ x_0 & x_{21} & \cdots & x_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ x_0 & x_{n1} & \cdots & x_{nk} \end{bmatrix} \quad A = \begin{bmatrix} A_1 \\ A_2 \\ \vdots \\ A_n \end{bmatrix} \quad \varepsilon = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{bmatrix}$$

Method of Least Squares

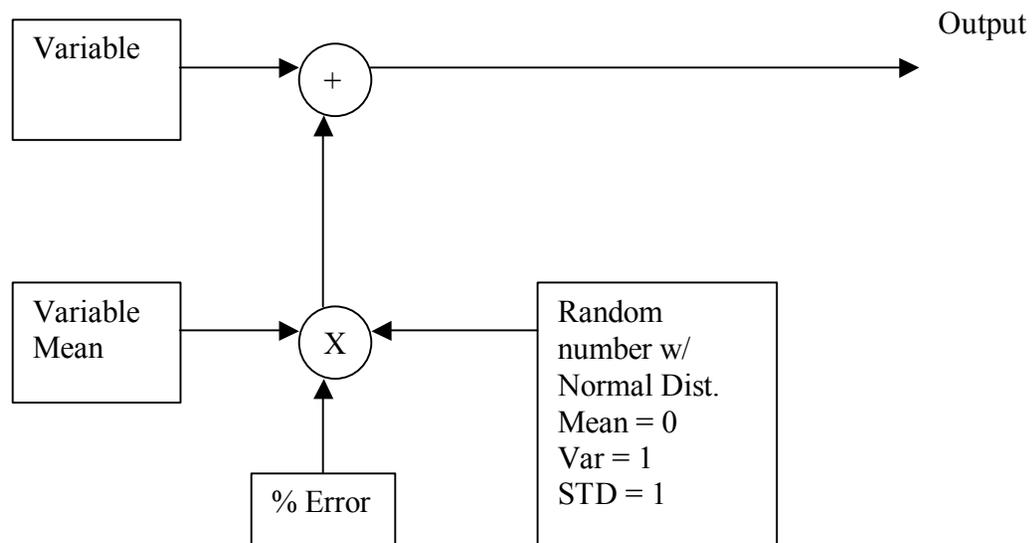
- A more compact expression is now possible

$$Y = X \cdot A + \varepsilon$$

- And hence the least squares solution is given by:

$$\hat{A} = (X' \cdot X)^{-1} X' \cdot Y$$

Introducing Instrumentation Error



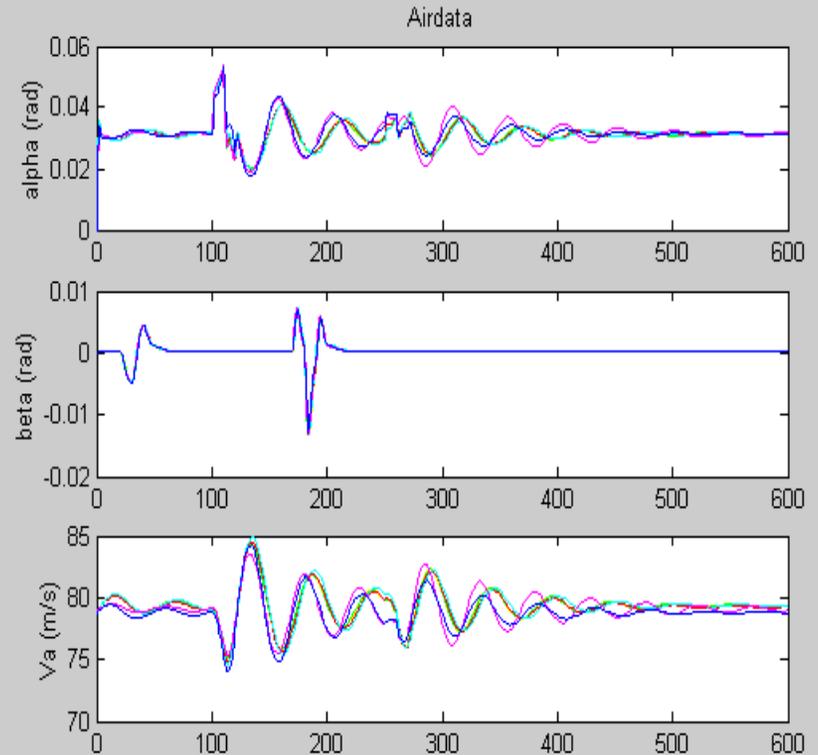
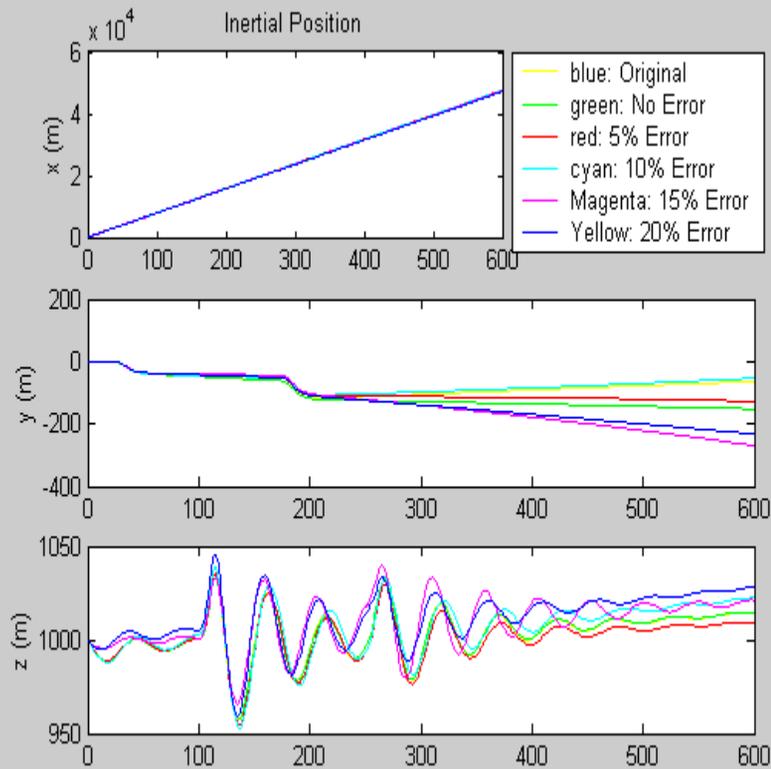
Results

Drag	CD0	CD(alpha)	CD(alpha^2)		
Org.	0.16	0.504	2.1175	0	0
0%	0.159929291	0.50391043	2.10434091	0	0
5%	0.160122879	0.516968066	1.978053264	0	0
10%	0.16099872	0.286702765	7.587558777	0	0
15%	0.156846126	0.907952248	-6.273333549	0	0
20%	0.168720167	0.156904935	6.405309376	0	0
Lift	CL0	CL(alpha)	CL(q/Va)	CL(de)	
Org.	1.0656	6.072	24.6	0.763	0
0%	1.0656283	6.072309878	24.59995508	0.763055945	0
5%	0.898915408	5.884932764	-36.14802631	-0.761129445	0
10%	0.889249704	6.378575947	-28.10416166	-0.686212668	0
15%	1.77378227	6.854781663	377.1349199	7.069507073	0
20%	0.654060465	4.827943992	31.10391869	-3.299369978	0
Sideforce	CY(beta)	CY(dr)			
Org.	-1.6	0.24	0	0	0
0%	-1.599901337	0.240000998	0	0	0
5%	-1.601807437	0.235356029	0	0	0
10%	-1.573137705	0.240223609	0	0	0
15%	-1.6696761	0.245773088	0	0	0
20%	-1.635071458	0.231498013	0	0	0

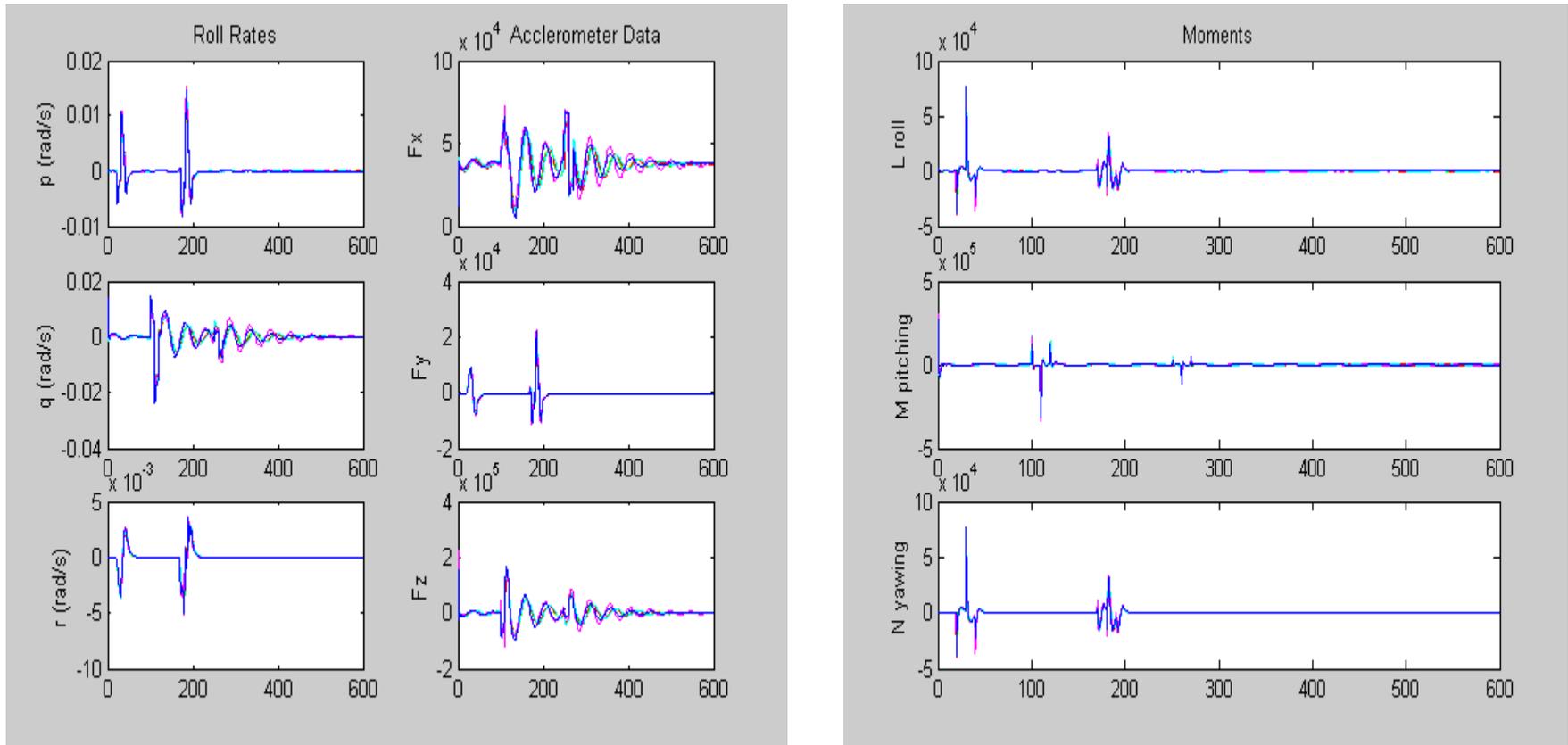
Results

Roll	CI(beta)	CI(p/Va)	CI(r/Va)	CI(da)	CI(dr)
Org.	-1.4	-72.6	33	-0.6	0.22
0%	-1.399990044	-72.60015177	32.99966499	-0.600000627	0.219998473
5%	-1.381409168	-71.27368113	32.90165795	-0.587945295	0.218413852
10%	-1.393691442	-72.83695566	32.04075897	-0.601878183	0.219292837
15%	-1.391820939	-69.85395133	34.46314938	-0.578464622	0.232403726
20%	-1.483364129	-77.01044315	36.08942334	-0.633080039	0.231437078
Pitch	CM0	CM(alpha)	CM(q/Va)	CM(de)	
Org.	-0.446	-2.15	-92.4424	-2.8673	0
0%	-0.446112948	-2.150498939	-92.43598582	-2.867238254	0
5%	-0.41866269	-2.117102684	-82.42520812	-2.615846801	0
10%	-0.426160771	-2.219389203	-87.06822821	-2.712622674	0
15%	-0.560141444	-2.265356318	-148.020325	-3.879641144	0
20%	-0.376329458	-1.975736201	-93.32965922	-2.190590635	0
Yaw	CN(beta)	CN(beta*alpha)	CN(p/Va)	CN(r/Va)	CN(dr)
Org.	1	-3.82	11.22	-75.9	-0.63
0%	0.999925037	-3.817521889	11.22000988	-75.89984888	-0.62999843
5%	1.001158747	-3.466322621	11.1528154	-77.49513786	-0.642749111
10%	0.965701165	-3.865166559	11.19698811	-71.23140125	-0.599203787
15%	1.154401414	-7.135800097	11.47061433	-81.23198181	-0.675784991
20%	1.091155546	-6.144717208	11.75292113	-78.096302	-0.638512996

Results



Results



Future Work

- Development of post-processing algorithms.
- The continued investigation of sensitivity to changes in aerodynamic coefficients, flight maneuvers, and noise models.
- Investigate alternative system identification tools.
- Verification of techniques with data collected from the Brumby.

References

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